International Trade with Sequential Production*

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Abstract

We develop a tractable general equilibrium model of international trade with firm heterogeneity and sequential production, based on the partial equilibrium model of Antràs and Chor (2013). The length of supply chains is endogenous, and in the autarky equilibrium more productive final-good producers are served by longer supply chains. International trade in final goods magnifies the differences in the length of supply chains between firms, with exporting firms being served by longer supply chains, and non-exporting firms being served by shorter supply chains than in autarky. We show that, for given model parameters, the gains from trade are larger than in the canonical Melitz model with Pareto-distributed productivities, since production along a sequential supply chain in the Antràs-Chor model is subject to external increasing returns to scale.

JEL classifications: F12, F15

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1 Introduction

The academic literature in international trade has recently started to pay attention to the fact that multi-stage production in many cases requires that the production process follows a certain sequence, exemplified by, among others, the papers of Harms et al. (2012), Baldwin and Venables (2013), Costinot et al. (2013) and Fally and Hillberry (2016). In an influential recent contribution to this literature, Antràs and Chor (2013) develop a property-rights model of the firm with sequential production stages, focusing on, and deriving results for, the optimal allocation of ownership rights along the value chain. In this paper, we embed the partial equilibrium model of Antràs and Chor (2013) into a standard general equilibrium framework featuring firm heterogeneity and costly international trade in final goods à la Melitz (2003), thereby making the model amenable to questions that have been traditionally of interest of trade economists, namely the effect of trade on domestic factor allocation and aggregate welfare. The model is developed in two steps. Our main results do not depend on the equilibrium allocation of ownership rights along the value chain, and we therefore derive these results in a benchmark framework, in which this allocation is exogenously fixed. Later we introduce the endogenous allocation of ownership rights, following Antràs and Chor (2013).

We consider an unbounded pool of prospective firms, intermediate-good suppliers (called "suppliers" in the following) and final-good producers. Prior to entry, firms are identical. To enter the market as a final-good producer, a firm pays an entry cost, thereby gaining access to a Melitz-style productivity lottery. To enter the market as a supplier, a firm pays a different entry cost, and afterwards is randomly assigned a type, represented by the relative position in the production chain. All suppliers have the same productivity. As in Antràs and Chor (2013), the production of final goods involves a large number of production stages. Each stage is sequentially performed by a specific supplier, and each supplier bargains with the final-good producer over the incremental contribution to total revenue generated at that stage. The equilibrium length of supply chains is determined by the condition that suppliers must be indifferent ex ante between joining different supply chains. Since aggregate profits are higher in supply chains linked to more productive final-good producers, the length of supply chains is increasing in the productivity of the respective final-good producer. Hence, in contrast to Antràs and Chor (2013), in our framework the extent of vertical specialisation is endogenously determined in general equilibrium.

In the open economy, as in Melitz (2003), the most productive final-good producers self-select into export markets, and the least productive firms exit. Due to additional sales in the export market, high-productivity final-good producers become more attractive to suppliers, and the supply chains linked to these firms increase in length. Non-exporting firms face more competition, which reduces their revenues, and the supply chains linked to these firms are therefore shorter than in autarky. We show that the productivity of an individual supply chain does not only depend only on the productivity of the final-good producer, but also on the length of this supply chain, in analogy to the role played by the range of intermediate inputs in Acemoglu et al. (2007). This implies that international trade in final goods has an effect on the productivity of individual supply chains, making the supply chains linked to exporting firms more productive, while the supply chains linked to non-exporting firms become less productive.

In order to find closed-form solutions for the share of exporting firms in our model, as well as for the gains from trade, we assume that the productivity distribution of final-good producers is Pareto. We then show that for identical underlying parameters the share of exporting firms is larger in our general equilibrium adaptation of the Antràs-Chor model than in the original Melitz model with Pareto-distributed productivities, and as a direct consequence the gains from trade are larger. We also show the same point by relating our analysis to the approach by Arkolakis et al. (2012), who devise a simple formula to compute the gains from trade based on observable trade data in a class of models that includes the Melitz model with Pareto distributed productivities. We show that for a given degree of openness, measured by the expenditure share on domestically produced goods as in Arkolakis et al. (2012), the gains from trade in the Antràs-Chor-Melitz model are larger than in the original Melitz model.

Our finding is related to a paper by Melitz and Redding (2014), who develop a model of sequential production in which international trade in intermediate inputs induces a reorganisation of production that raises domestic productivity in final-good production. The authors show that in such a model the welfare gains from trade can become arbitrarily large as the exogenous number of production stages becomes arbitrarily large, since gains from trade exist at each production stage. Our key result is complementary to Melitz and Redding (2014) since we consider the effect of trade in final goods, and the additional welfare gains from trade in our paper result from induced changes in the length of supply chains in the open economy, which induce a change in the productivity at the level of each supply chain. The gains from trade in our model cannot be infinitely large, since the length of supply chains is determined endogenously rather than set parametrically.

The remainder of this paper is structured as follows. Section 2 derives our key general equilibrium results in a simplified general equilibrium version of the Antràs-Chor model, which treats suppliers' bargaining power, and hence firm organisation, as exogenous. Section 3 revisits the original mechanism for optimal firm organisation from Antràs and Chor (2013) and Alfaro et al. (2016) in our general equilibrium setting. We show that our earlier derived general equilibrium results are robust to this change. Section 4 concludes.

2 A Benchmark Model with Exogenous Firm Organisation

In this section, we develop a benchmark version of our general equilibrium framework with firm heterogeneity and sequential. In this first step, we treat firm organisation as exogenous in order to simplify the analysis. As we show below in Section 3, all our general equilibrium results survive the transition to a richer framework that introduces the original mechanism for optimal firm organisation from Antràs and Chor (2013) and Alfaro et al. (2016).

2.1 Model Basics

Preferences of a representative consumer over varieties of the final good are given by the CES utility function

$$U = \left[\int_{v \in V} x(v)^{\rho} \, dv \right]^{\frac{1}{\rho}}, \qquad 0 < \rho \equiv \frac{\sigma - 1}{\sigma} < 1 \tag{1}$$

where $x(v)$ is the consumption of variety v, V is the set of available varieties, and σ is the constant elasticity of substitution. Denoting the price of variety v by $p(v)$, the ideal price index is

$$P = \left[\int_{v \in V} p(v)^{1-\sigma} \, dv \right]^{\frac{1}{1-\sigma}}$$

with $R = PX$, where $X \equiv U$ is considered as an aggregate consumption good, and R is total expenditure. The demand function for each variety v follows as

$$x(v) = A \left[p(v) \right]^{-\frac{1}{1-\rho}}, \tag{2}$$

where demand shifter $A \equiv P^{\sigma-1} R$ is determined in general equilibrium but treated parametrically by firms.

Following Antràs and Chor (2013), we assume that the production of final goods involves a large number of sequential production stages, indexed by M. Each stage is performed by a specific supplier, and each supply chain serves one final-good producer. Labour L is the only factor of production, with the wage w determined in a perfectly competitive labour market. Labour is chosen as the *numéraire*, and hence the wage is normalised to 1. The final-good producer provides a supply-chain-level public good in the form of its technology to all suppliers, and incurs a fixed cost f.[1] The productivity of the final-good producer in this supply chain is denoted by φ. Each supplier at stage $m \in [0, M]$ decides to hire $l(m)$ units of labour. We also assume that each supplier just engages in one supply chain, therefore supply chains are

[1] This idea is analogous to firm-level assets "that are applied in one part of the firm can also be applied in another part" (Navaretti et al., 2004); or to managers who "are able to solve a wider range of the problems their team confronts in production" (Antràs et al., 2006).

independent from each other.

The constant-elasticity of substitution production function of the final goods with parameters $\alpha \in (0,1)$ and $s \geq 0$ is given by

$$q(M,\varphi) = \varphi M^{s+1-1/\alpha} \left(\int_0^M l(i)^\alpha I(i)\, di \right)^{1/\alpha},$$

where i indexes the stage of production. $I(i) = 1$ if input i is produced after the inputs in all previous stages $i' < i$ have been produced, otherwise $I(i) = 0$. Following Benassy (1998), we call s the degree of *returns to specialisation.* Our production function collapses to the one in Antràs and Chor (2013) if we set $s = (1-\alpha)/\alpha$. As in Acemoglu et al. (2007), allowing for $s \neq (1-\alpha)/\alpha$ makes it possible to disentangle the elasticity of substitution between inputs in each stage and the elasticity of output with respect to the number of production stages in one supply chain. We assume $s < (1-\rho)/\rho$. As we show below, this assumption ensures that the elasticity of output with respect to the number of production stages is not too large, which in turn is crucial for a non-degenerate equilibrium of our model. The production process at each stage is coordinated and served along the supply chain by a final-good producer with productivity φ. Firms with higher productivity φ coordinate and serve supply chains more efficiently, thereby increasing output, ceteris paribus.

Imposing goods market equilibrium, and using Eq. (2) to substitute for p, firm-specific revenues $r(v)$ can be written as

$$r(v) = A^{1-\rho} [q(v)]^\rho . \tag{3}$$

Since all firm-specific variables depend on φ, we will use this variable instead of v to denote a particular final-good producer from now on. In a supply chain belonging to a final-good producer with productivity φ, the revenue up to stage m is given by

$$r(M,m,\varphi) = A^{1-\rho}\varphi^{\rho}M^{(s+1-1/\alpha)\rho}\left(\int_0^m [l(i)]^{\alpha} I(i)\,di\right)^{\rho/\alpha}$$
$$= A^{1-\rho}q(M,m,\varphi)^{\rho} \tag{4}$$

where $q(M,m,\varphi) = \varphi M^{s+1-1/\alpha}\left(\int_0^m l(i)^{\alpha} I(i)\,di\right)^{1/\alpha}$ denotes production up to stage m. The incremental revenue generated by a supplier at stage m, $r'(M,m,\varphi)$, can be written as a function of $r(M,m,\varphi)$ by differentiating Eq. (4) and applying Leibniz's rule:

$$r'(M,m,\varphi) = \frac{\rho}{\alpha}\left[A^{1-\rho}\varphi^{\rho}M^{(s+1-1/\alpha)\rho}\right]^{\frac{\alpha}{\rho}} r(M,m,\varphi)^{\frac{\rho-\alpha}{\rho}}\, l(m)^{\alpha} \tag{5}$$

As pointed out by Antràs and Chor (2013), if $\rho-\alpha > 0$, upstream suppliers' investments increase the incremental value made by stage m, and supplier investments are sequential complements; if $\rho - \alpha < 0$, upstream suppliers' investments reduce the incremental contribution made by the supplier at stage m, and supplier investments are sequential substitutes. Although our general equilibrium results do not depend on whether supplier investments are sequential complements or substitutes, we refer to $\rho > \alpha$ as the complements case and to $\rho < \alpha$ as the substitutes case, following Antràs and Chor (2013).

We now summarise the timeline of the game played by final-good producers and the continuum of suppliers:

1. There is an unbounded pool of prospective firms. In order to enter the market as a final-good producer, a firm has to pay an entry cost f_e. In order to enter the market as a supplier, a firm has to pay an entry cost f_m. Both fixed costs are paid in units of labour.

2. Suppliers draw their type, which is the relative position $h \equiv m/M$ they will assume in any supply chain they join, from a uniform distribution. They all have the same productivity, normalised to 1. Final-good producers draw their productivity φ from a distribution $G(\varphi)$. After realisation of the productivity draw, a final-good producer decides to start

production, which requires an additional fixed cost f, or to quit the market. There is no fixed cost for suppliers.

3. Supply chains are formed by matching between final-good producers and suppliers. The latter can choose which supply chain to join, and therefore equilibrium profits for each relative position h are equalised across supply chains. Production then takes place sequentially. A supplier at stage m treats parametrically the value of the unfinished product handed over by the upstream supplier, $r(M, m, \varphi)$, and decides how much labour to hire for production in order to maximise profits.

4. As in Antràs and Chor (2013), the supplier at stage m bargains with the final-good producer over the incremental contribution to total revenue generated at that stage. We assume that the bargaining power is $1-\beta$ for all suppliers, $0 < \beta < 1$. Hence, each supplier at stage m receives a share $1-\beta$ of the incremental contribution $r'(M, m, \varphi)$.

5. All final-good producers and all suppliers die after one period.

2.2 Closed Economy Equilibrium

We analyse the subgame perfect equilibrium of the game described above. We start by considering the situation under which a supplier at stage m is matched to a final-good producer with productivity φ. The payment this supplier receives from the final-good producer is given by $(1-\beta)\, r'(M, m, \varphi)$, and the profit maximising employment level is the solution to

$$\max_{l(m,\varphi)} \left\{ (1-\beta)\, \frac{\rho}{\alpha} \left[A^{1-\rho} \varphi^{\rho} M^{(s+1-1/\alpha)\rho} \right]^{\frac{\alpha}{\rho}} r\,(M, m, \varphi)^{\frac{\rho-\alpha}{\rho}}\, l\,(m, \varphi)^{\alpha} - l(m, \varphi) \right\} \tag{6}$$

The optimal level of employment (and wage cost) of the supplier at stage m follows as:

$$l\,(m, \varphi) = \left[(1-\beta)\, \rho \left(A^{1-\rho} \varphi^{\rho} M^{(s+1-1/\alpha)\rho} \right)^{\frac{\alpha}{\rho}} r\,(M, m, \varphi)^{\frac{\rho-\alpha}{\rho}} \right]^{\frac{1}{1-\alpha}} \tag{7}$$

We get the standard result with Cobb-Douglas production functions that a constant share $1-\alpha$ of the supplier's revenue is profit, with the remainder being wage cost. Plugging the expression from Eq. (7) into Eq. (5) and solving the resulting differential equation leads to an expression for the revenue accrued up to stage m:

$$r\left(M,m,\varphi\right)=A\left[m\left(\frac{1-\rho}{1-\alpha}\right)\right]^{\frac{\rho(1-\alpha)}{(1-\rho)\alpha}}\left[\rho\left(1-\beta\right)\varphi M^{s+1-1/\alpha}\right]^{\frac{\rho}{1-\rho}} \tag{8}$$

It is easily checked that $l\left(m,\varphi\right)$ is positive for all m as long as $0<\beta<1$. This implies that each supplier has an incentive to follow the proper sequential production, hence $I\left(m\right)=1$ for all m. The revenue of a final-good producer with productivity φ follows from setting $m=M$ in Eq. (8):

$$r\left(M,\varphi\right)=\left(1-\beta\right)^{\frac{\rho}{1-\rho}}\Omega A\left(M^{s}\varphi\right)^{\frac{\rho}{1-\rho}},\quad\text{with}\quad\Omega\equiv\left(\frac{1-\rho}{1-\alpha}\right)^{\frac{\rho(1-\alpha)}{(1-\rho)\alpha}}\rho^{\frac{\rho}{1-\rho}}, \tag{9}$$

where Ω collects parameters.

Looking at the micro-structure underlying these aggregates, we can use Eqs. (7) and (8), to express employment of an individual supplier at stage m as a function of φ and m:

$$l(m,M,\varphi)=Z(A,\rho,\alpha,\beta)\varphi^{\frac{\rho}{1-\rho}}\left[M(\varphi)\right]^{\frac{(1+s-1/\alpha)\rho}{1-\rho}}m^{\frac{\rho-\alpha}{\alpha(1-\rho)}}, \tag{10}$$

where $Z(\cdot)$ collects terms, and demand shifter A and the mass of suppliers M are endogenous. It follows directly from Eq. (10) that supplier-level employment varies along a supply chain with constant elasticity $(\rho-\alpha)/[\alpha(1-\rho)]$, and hence downstream suppliers along a given supply chain have higher employment levels than upstream suppliers if and only if $\rho>\alpha$, i.e. if intermediate goods are sequential complements.

Suppliers know their relative position h before deciding on which supply chain to join, and therefore equilibrium must fulfill the *supplier indifference condition* (SIC) that profits for a supplier with relative position h must be equalised across supply chains. Since suppliers' profits are proportional to their wage costs, we can equivalently state the SIC in terms of wage costs. To this end, we simply rewrite Eq.(10) in terms of h to get

$$l(h, M, \varphi) = Z(A, \rho, \alpha, \beta)\varphi^{\frac{\rho}{1-\rho}}[M(\varphi)]^{\frac{1-(s+1)\rho}{\rho-1}} h^{\frac{\rho-\alpha}{\alpha(1-\rho)}}, \tag{11}$$

Using Eq. (11), the SIC says that for all pairs of productivities φ_1 and φ_2

$$\frac{l(h, M(\varphi_1), \varphi_1)}{l(h, M(\varphi_2), \varphi_2)} = 1 \tag{12}$$

must hold. It is easily checked that the parameter constraint $s < (1-\rho)/\rho$ introduced above ensures that $l(h, M, \varphi)$ is decreasing in M, which is necessary and sufficient for Eq. (12) to have a solution. If the parameter constraint were violated, all suppliers would have an incentive to join the supply chain serving the most productive final-good producer, and just one supply chain would survive in the economy.[2] Combining Eqs. (11) and (12), we get

$$\frac{M(\varphi_1)}{M(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\varepsilon}, \qquad \text{with} \qquad \varepsilon \equiv \frac{\rho}{1-(s+1)\rho}. \tag{13}$$

Thereby, ε is the elasticity of the supply chain length with respect to the productivity of the final-good producer to which the supply chain belongs. The assumption $s < (1-\rho)/\rho$ also implies $\varepsilon > 0$, and hence more productive final-good producers are served by longer supply chains.

It follows from Eqs. (9) and (13) that revenues of final-good producers are also increasing in firm-level productivity with constant elasticity ε. This elasticity exceeds the corresponding elasticity of revenues with respect to productivity in the Melitz model, given by $\rho/(1-\rho)$, provided that the returns to specialisation s are strictly positive. This is what we consider to be the normal case in the following, and we will exclude the borderline case $s = 0$ unless mentioned

[2] As noted above, the production function we use collapses to the standard CES function used by Antràs and Chor (2013) for $s = (1-\alpha)/\alpha$. The concavity condition in this case would become $\rho < \alpha$, which is the condition for the substitutes case. Hence, a benefit of the more flexible CES function we use is that it allows us to also look at the sequential complements case in general equilibrium.

specifically. This magnified effect relative to Melitz (2003) is due to the fact that the sequential production structure of Antràs and Chor (2013) is subject to positive returns to specialisation (unless $s = 0$), while firm-level production in Melitz (2003) is not.

We now turn to solving the full general equilibrium of our model. The profit of a final-good producer with productivity φ is given by $\pi\left(M,\varphi\right) = \beta r\left(M,\varphi\right) - f$, and the zero cutoff profit condition, implicitly defined by $\pi(\varphi^{*}) = 0$, follows as

$$r[M(\varphi^{*}),\varphi^{*}] = \frac{f}{\beta} \tag{14}$$

Analogously, the free entry condition for suppliers requires that their ex-ante expected profit is equal to the entry cost f_m. With $r[M(\varphi),\varphi]$ and $M(\varphi)$ increasing in φ with the same elasticity, average revenues per supplier (and therefore ex-ante expected profits) are equalised across supply chains, and we can therefore write down an identical free entry condition in terms of any supply chain φ. But in order to match the free entry condition for final-good producers, it is particularly informative to write down the free entry condition for suppliers joining supply chain φ^{*}. It requires that the total profits accrued by suppliers in the supply chain serving the final-good producer with productivity φ^{*} equals the combined entry cost of all suppliers in this supply chain:

$$(1-\alpha)(1-\beta)r[M(\varphi^{*}),\varphi^{*}] = M(\varphi^{*})f_m \tag{15}$$

It is immediate that Eqs. (14) and (15) can be solved for the length of the supply chain organised by the marginal final-good producer with productivity φ^{*}:

$$M\left(\varphi^{*}\right) = \frac{f\left(1-\alpha\right)\left(1-\beta\right)}{f_m\beta} \tag{16}$$

Intuitively, the length of the supply chain serving the marginal producer is high if revenues of the marginal producer are high (true if f is high), the share of revenues going to suppliers is high (β is low), the share α of suppliers' revenues going to workers is low, and the fixed cost for suppliers f_m is low.

The average revenue per supplier in any supply chain φ follows directly from Eqs. (14) and (16) as

$$\frac{(1-\beta)r[M(\varphi),\varphi]}{M(\varphi)} = \frac{f_m}{(1-\alpha)}, \tag{17}$$

and the average employment per supplier in any supply chain φ is given by

$$\frac{\int_0^{M(\varphi)} l(m,\varphi)dm}{M(\varphi)} = \left(\frac{\alpha}{1-\alpha}\right) f_m \tag{18}$$

We assume that the productivity of final-good producers is drawn from a Pareto distribution $G(\varphi) = 1 - \varphi^{-k}$, with $k > \varepsilon$ in order to ensure that the relevant model variables have finite means. Using Eq. (13), the average length of supply chains then follows as

$$\overline{M} = \int_{\varphi^*}^{\infty} M(\varphi) \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi = \Theta M(\varphi^*) \qquad \text{with} \qquad \Theta \equiv \frac{k}{k-\varepsilon}. \tag{19}$$

In analogy to Eq. (19), and using the zero cutoff profit condition in Eq. (14), the final-good producer's ex-ante expected profit $\overline{\pi}$ can be written as

$$\overline{\pi} = \int_{\varphi^*}^{\infty} \pi(\varphi) \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi = (\Theta - 1) f \tag{20}$$

The free entry condition for final-good producers takes the standard form

$$(\varphi^*)^{-k}\,\overline{\pi} = f_e, \tag{21}$$

and together Eqs. (20) and (21) allow us to write the cutoff productivity φ^* as a function of model parameters:

$$\varphi^* = \left[\frac{(\Theta-1) f}{f_e}\right]^{\frac{1}{k}} \tag{22}$$

Notably, the cutoff productivity for final-good producers as derived in Eq. (22) is strictly larger, for given values of parameters f, f_e, k, and σ than the cutoff productivity φ_M^* in the standard

Melitz model with Pareto-distributed productivity. In this model, the cutoff productivity is given by

$$\varphi_M^* = \left[\frac{(\Theta_M - 1) f}{f_e}\right]^{\frac{1}{k}},$$

with $\Theta_M \equiv k/(k - \sigma + 1)$, which is strictly smaller than Θ as long as s is strictly positive. The intuition is straightforward: The returns to specialisation along a supply chains benefit disproportionately the final-good producers organising longer supply chains, which are the more productive firms. As a consequence, the least productive final-good producers that would have near-zero profits in the Melitz model are not profitable in an otherwise identical model with sequential production à la Antràs and Chor (2013).

With labour the only factor of production and free entry, we have $R = L$, as in Melitz (2003). Therefore the aggregate revenue of final-good producers is equal to L. Since it follows from Eq. (20) that the average revenue of final-good producers equals $\Theta f/\beta$, the number of final-good producers is given by $N = \beta L/(\Theta f)$, and the total number of suppliers follows as

$$\overline{M}N = \frac{(1-\beta)(1-\alpha)}{f_m} L. \tag{23}$$

Recalling the property of the ideal price index, and using the zero cutoff profit condition for final-good producers, Eq. (14), welfare per worker is given by

$$W = P^{-1} = \left(\frac{(1-\beta)^{\frac{\rho}{1-\rho}} \beta \Omega L}{f}\right)^{\frac{1-\rho}{\rho}} [M(\varphi^*)]^s \varphi^*. \tag{24}$$

2.3 Open Economy Equilibrium

We now look at the trade equilibrium in a world of two identical countries, Home and Foreign. Exporting of final goods involves two types of costs: f_x for each firm that exports and variable iceberg transport cost, represented by parameter $\tau > 1$. Since our interest lies in the effects of trade in final goods, we do not model trade costs for intermediate goods, and hence we do not take a stand on whether supply chains are organised nationally or globally. But as in the closed economy, we assume that each supplier just engages in one supply chain.

With identical countries, an exporting firm has foreign revenue that is a multiple $\theta \equiv \tau^{\rho/(\rho-1)} < 1$ of its domestic revenue. The marginal exporting firm with productivity φ_x^* needs to be indifferent between exporting and non-exporting, and therefore its gain in operating profits from exporting is equal to the fixed cost of exporting f_x. Formally, the indifference condition can be written as

$$\beta \{(1+\theta) r[M_x(\varphi_x^*), \varphi_x^*] - r[M_d(\varphi_x^*), \varphi_x^*]\} = f_x, \tag{25}$$

where $M_x(\varphi)$ and $M_d(\varphi)$ denote the length of a supply chain attached to an exporting firm and to a non-exporting firm, respectively, $r(\cdot)$ refers to revenue generated in the Home market, and in analogy to Eq. (9), we have

$$r[M_i(\varphi_x^*), \varphi_x^*] = (1-\beta)^{\frac{\rho}{1-\rho}} \Omega A \{[M_i(\varphi_x^*)]^s \varphi_x^*\}^{\frac{\rho}{1-\rho}}, \qquad i = x, d.$$

In direct analogy to Eq. (11), the wage cost of a supplier at relative position h in the supply chain serving an exporting final-good producer with productivity φ is given by

$$l_x(h, M_x, \varphi) = (1+\theta) Z(A, \rho, \alpha, \beta) \varphi^{\frac{\rho}{1-\rho}} [M_x(\varphi)]^{\frac{1-(s+1)\rho}{\rho-1}} h^{\frac{\rho-\alpha}{\alpha(1-\rho)}},$$

while the respective expression for a non-exporting final-good producer is still given by Eq. (11). Hence, employment along a given supply chain varies with elasticity $(\rho-\alpha)/[\alpha(1-\rho)]$, unchanged from the closed economy, for both exporting and non-exporting firms.

M_d and M_x are linked by the *cross-regime SIC*, according to which a supplier at relative position h must be ex ante indifferent between joining a production chain linked to an exporting firm or one linked to a non-exporting firm of the same productivity:

$$\frac{l_x[h, M_x(\varphi), \varphi]}{l_d[h, M_d(\varphi), \varphi]} = 1$$

Substituting for firm-level wage costs, the cross-regime SIC implies

$$\frac{M_x(\varphi)}{M_d(\varphi)} = (1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}}. \tag{26}$$

Hence, exporting final-good producers have longer supply chains, ceteris paribus.

Combining the indifference condition for the marginal exporting firm, Eq. (25), the cross-regime SIC, Eq. (26), the zero profit condition for the marginal final-good producer, Eq. (14), and the within-regime SIC, Eq. (13), we can write the ratio of the two cutoff productivities φ_x^* and φ_d^* as a function of model parameters only:[3]

$$\frac{\varphi_x^*}{\varphi_d^*} = \left\{ \frac{f_x}{f\left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right]} \right\}^{\frac{1}{\varepsilon}} \equiv \xi, \tag{27}$$

where as usual we assume that f_x/f is sufficiently large to guarantee that φ_x^* is strictly larger than φ_d^*. With the assumed Pareto distribution of productivities, this directly translates into a solution for the share of exporting firms χ:

$$\chi = \frac{1 - G(\varphi_x^*)}{1 - G(\varphi_d^*)} = \xi^{-k}$$

The share of exporting firms in our model is larger, for given parameter values τ, f_x, f, k, and σ than the corresponding measure χ_M in the Melitz model with Pareto-distributed productivities, which is given by

$$\chi_M = \left(\frac{f_x}{f\theta}\right)^{-\frac{k(1-\rho)}{\rho}}$$

The inequality $\chi \geq \chi_M$ is strict whenever s is strictly positive.

Ex ante average profits $\bar{\pi}$ in the open economy are a weighted average of average domestic profits $\bar{\pi}_d$ and average export profits $\bar{\pi}_x$. As we show in the Appendix, we can write average

[3]The required steps are as follows. First, one can use Eq. (26) to substitute for $M_x(\varphi_x^*)$ in Eq. (25). Combining the resulting expression with Eq. (14) gives a relationship between $M_d(\varphi_x^*)/M_d(\varphi_d^*)$ and φ_x^*/φ_d^*. Substituting for $M_d(\varphi_x^*)/M_d(\varphi_d^*)$ from Eq. (13) yields the desired result.

profits as a function of model parameters and χ:

$$\bar{\pi} = \bar{\pi}_a \left(1 + \chi \frac{f_x}{f}\right), \tag{28}$$

with subscript a denoting autarky. Together with the free entry condition for final-good producers, unchanged from the autarky case and hence given by Eq. (21), we can solve for the domestic cutoff productivity φ_d^*:

$$\varphi_d^* = \varphi_a^* \left(1 + \chi \frac{f_x}{f}\right)^{\frac{1}{k}} \tag{29}$$

Hence, we have the standard result that trade increases the cutoff productivity relative to autarky.

While trade leads to a higher cutoff productivity, the length of the supply chain managed by the marginal firm, $M(\varphi_d^*)$, is unaffected by trade, and therefore it is still given by Eq. (16). Similarly, average revenue and average employment per supplier within a supply chain are still given by Eqs. (17) and (18), respectively. Hence, as in autarky, a crucial role is played by the differences across firms in the length of supply chains managed by them. For a non-exporting firm with given productivity φ the effect of the transition from autarky to trade on the length of the supply chain is given by

$$\frac{M_d(\varphi)}{M_a(\varphi)} = \left(\frac{\varphi_d^*}{\varphi_a^*}\right)^{-\varepsilon}, \tag{30}$$

while for an exporting firm the effect is given by

$$\frac{M_x(\varphi)}{M_a(\varphi)} = (1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} \left(\frac{\varphi_d^*}{\varphi_a^*}\right)^{-\varepsilon}, \tag{31}$$

It follows immediately from Eqs. (29) and (30) that $M_d(\varphi)/M_a(\varphi) < 1$, and therefore the transition to trade decreases the length of supply chains managed by firms that do not export in the open economy. Furthermore, we prove in the Appendix that $M_x(\varphi)/M_a(\varphi) > 1$, and therefore the transition to trade leads to longer supply chains for those firms that do export in the open economy. We therefore have the following proposition:

Proposition 1 *International trade in final goods leads to longer exporting supply chains, but to shorter non-exporting supply chains.*

Proof. See the Appendix. ■

In analogy to Eq. (28), trade affects average revenues $\bar{r}$ and the average supply chain length $\overline{M}$ according to $\bar{r}/\bar{r}_a = \overline{M}/\overline{M}_a = 1 + \chi f_x/f$. And with $N = \beta L/\bar{r}$, it follows directly that $N/N_a = (1+\chi f_x/f)^{-1}$. Hence, the total number of suppliers $\overline{M}N$ stays constant in the transition from the closed to the open economy.

Welfare per worker can be calculated in analogy to Eq. (24). The gains from trade are given by

$$\frac{W}{W_a} = \frac{\varphi_d^*}{\varphi_a^*} = \left(1 + \chi \frac{f_x}{f}\right)^{\frac{1}{k}} \tag{32}$$

In contrast, the Melitz (2003) model with Pareto distributed productivities gives gains from trade equal to

$$\frac{W_M}{W_{Ma}} = \left(1 + \chi_M \frac{f_x}{f}\right)^{\frac{1}{k}}$$

Since, as shown above, $\chi > \chi_M$ for given parameter values τ, f, f_x, σ, and k, the gains from trade are larger, ceteris paribus, in the Antràs-Chor-Melitz model developed here than in Melitz (2003).

Alternatively, we can make the same point by looking at the gains from trade using the approach by Arkolakis et al. (2012), who show that the gains from trade in the Melitz model with Pareto-distributed productivities can be written as $\lambda_d^{-1/k}$, where λ_d is the expenditure share going to domestic products in the trade equilibrium. In the Appendix, we compute the share λ_d for the Antràs-Chor-Melitz model, and we show that it is larger than $(1 + \chi f_x/f)^{-1}$. In light of Eq. (32), this implies that the gains from trade are larger than in the Melitz model with Pareto-distributed productivities, conditional on observed trade data.[4] On the other hand, unlike such gains in the sequential production model in Melitz and Redding (2014), which can

[4]Echoing our earlier results, in the absence of external increasing returns (i.e. with $s = 0$) λ_d equals $(1 + \chi f_x/f)^{-1}$ in our model, and the gains from trade can be computed using the formula from Arkolakis et al. (2012).

be arbitrarily large if the number of production stages becomes arbitrarily large, the gains from trade in our model can not be infinite. This is simply because in our model the number of suppliers in each supply chain is endogenous.

One way to see what is responsible for the magnified gains from trade in the Antràs-Chor-Melitz model is to look at the productivity at the level of a single supply chain, defined as the output of the final-good firm divided by the total number of labour units employed in the supply chain organised by this firm:

$$\Phi(M,\varphi) = \frac{q(M,\varphi)}{\int_0^M l(m,\varphi)\,dm}$$

From Eqs. (3) and (9), firm-level output is given by

$$q(M,\varphi) = \left[(1-\beta)^{\frac{\rho}{1-\rho}}\,\Omega\right]^{\frac{1}{\rho}} A\,(M^s\varphi)^{\frac{1}{1-\rho}}\,,$$

and from maximisation program (6), we know that supply-chain level employment is equal to

$$\int_0^M l(m,\varphi)\,dm = \alpha(1-\beta)(1-\beta)^{\frac{\rho}{1-\rho}}\,\Omega A\,(M^s\varphi)^{\frac{\rho}{1-\rho}}$$

Therefore

$$\Phi(M,\varphi) = \frac{\rho}{\alpha}\left(\frac{1-\rho}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}} M^s\varphi$$

The productivity of an individual supply chain depends on its final-good producer's productivity φ and the length of this supply chain M. As in Acemoglu et al. (2007), this result links to the well-established representation of technology: the range of intermediate inputs. In our model, a more productive supply chain involves more suppliers as well as a deeper vertical specialisation. In combination with Proposition 1, we therefore have the following corollary:

Corollary 1 *International trade in final goods increases the average productivity of suppliers in supply chains organised by exporting firms and decreases the average productivity of suppliers in supply chains organised by non-exporting firms.*

As in Melitz (2003), international trade in the Antràs-Chor-Melitz model leads to a reallocation of workers towards more productive firms. But the positive welfare effect of this process is stronger than in Melitz (2003) since the productivity of individual supply chains managed by exporting firms increases in the process, while the productivity of supply chains managed by non-exporting firms decreases.

3 The Endogenous Organisation of Supply Chains

In the analysis so far, we have developed a framework that uses as one of its building blocks a version of the Antràs-Chor model that – in order to increase transparency of our analysis – is simplified in one crucial dimension: the share of the incremental revenue at each production stage that is appropriated by the final-good producer is not endogenous as in Antràs and Chor (2013) and Alfaro et al. (2016), but exogenously equal to parameter β. We have therefore eliminated from our framework the choice of a final-good producer whether or not to integrate suppliers along the supply chain, thereby influencing the share of the incremental revenue it can appropriate at each stage. We now introduce this feature of the original Antràs-Chor model and show that it leaves the general equilibrium results derived in our benchmark model qualitatively unchanged.

As in Antràs and Chor (2013), the final-good producer φ needs to decide whether or not to integrate a supplier m. Following the idea of the property-rights approach, the final-good producer receives a share β_v of the incremental revenue at stage m if the supplier at that stage is integrated, while it obtains a share $\beta_o < \beta_v$ of the incremental revenue if this supplier is not integrated.[5] Denoting by $\beta(m)$ the incremental revenue share accruing to the final-good producer when bargaining with the supplier at stage m, and noting that $\beta(m) \in \{\beta_v, \beta_o\}$, Eq. (6) can be rewritten as

$$\max_{l} \left\{ [1-\beta(m)] \frac{\rho}{\alpha} \left[A^{1-\rho} \varphi^{\rho} M^{(s+1-1/\alpha)\rho} \right]^{\frac{\alpha}{\rho}} r(M, m, \varphi)^{\frac{\rho-\alpha}{\rho}} l(m, \varphi)^{\alpha} - l(m, \varphi) \right\} \tag{33}$$

[5] The property-rights approach was developed by the seminal work of Grossman and Hart (1986).

Solving Eq. (33), following the steps outlined in Alfaro et al. (2016), we obtain the optimal choice of labour employment $l\left[m, \beta\left(m\right)\right]$.[6] Plugging this expression into the marginal contribution function $r'\left(M, m, \varphi\right)$ yields a separable differential equation. Solving it, we can get the total revenue accrued along a supply chain φ in the closed economy, analogous to Eq. (9) from our benchmark model:

$$r\left(M, \varphi\right) = \Omega A\left(\varphi M^{s+1-1/\alpha}\right)^{\frac{\rho}{1-\rho}}\left[\int_0^M\left[1-\beta\left(i\right)\right]^{\frac{\alpha}{1-\alpha}} di\right]^{\frac{\rho(1-\alpha)}{(1-\rho)\alpha}} \tag{34}$$

The final-good producer chooses the path of $\beta\left(i\right) \in \{\beta_v, \beta_o\}$ to maximise its operating profits $\int_0^M \beta\left(i\right) r'\left(M, i, \varphi\right) di$.

The key firm-level results in Antràs and Chor (2013) and Alfaro et al. (2016) also hold in our framework. In particular, for a final-good producer φ served by a supply chain of length $M(\varphi)$ there exist thresholds $m_c^*(\varphi) \in (0, M(\varphi)]$ and $m_s^*(\varphi) \in (0, M(\varphi)]$ separating the stages that are outsourced from those that are integrated in the complements case and the substitutes case, respectively. With sequential complements, all production stages before $m_c^*(\varphi)$ are outsourced, with the remainder being integrated, while in the case of sequential substitutes all stages before $m_s^*(\varphi)$ are integrated, with the remainder being outsourced.

Following the same calculations as in Alfaro et al. (2016), we can show in the Appendix that the value of these thresholds can be written in our model as

$$\frac{m_k^*(\varphi)}{M(\varphi)} = h_k^* = \Psi_k\left(\beta_v, \beta_o, \alpha, \rho\right) \text{ with } k = c, s$$

where $\Psi_k\left(\beta_v, \beta_o, \alpha, \rho\right)$ collects terms involving β_v and β_o, and is independent of φ. This means that only the relative position of a supplier matters for the final-good producer's integration decision. Plugging the expression for m_k^* into Eq. (34), we can rewrite total revenue along a

[6] A detailed derivation can be found in the Appendix A-1 of Alfaro et al. (2016), and it is also contained in our Appendix section. Notice that in our model the length of an individual supply chain is M rather than the normalised length, 1. Unlike in Alfaro et al. (2016), we assume that suppliers all have the same productivity $c\left(m\right)$, normalised to 1, and that there is no asymmetry $\psi\left(m\right)$ in the marginal product of different inputs' investment, thus $\psi\left(m\right) = c\left(m\right) = 1$ in our model.

supply chain as (see Appendix):

$$
\begin{aligned}
r(M,\varphi) &= \Omega A \left(\varphi M^{s+1-1/\alpha}\right)^{\frac{\rho}{1-\rho}} \left[\int_0^{h_k^* M} [1-\beta(i)]^{\frac{\alpha}{1-\alpha}}\, di + \int_{h_k^* M}^{M} [1-\beta(j)]^{\frac{\alpha}{1-\alpha}}\, dj\right]^{\frac{\rho(1-\alpha)}{(1-\rho)\alpha}} \\
&= \Omega A \left(M^s \varphi\right)^{\frac{\rho}{1-\rho}} \Lambda_k(\beta_v, \beta_o, \alpha, \rho)
\end{aligned}
$$

with $\beta(i) = \beta_o$ and $\beta(j) = \beta_v$ in the complements case, and with $\beta(i) = \beta_v$ and $\beta(j) = \beta_o$ in the substitutes case. Λ_k is another parameter collecting terms involving β_v and β_o.

We can easily check that the SIC and zero profit condition work as in our benchmark model. All previous results are also valid in this model extension. In particular, the gains from trade W/W_a are the same as in our benchmark model. International trade has no effect on the threshold relative position h_k^*, and therefore the analysis in Section 2.3 still applies.

4 Conclusions

In this paper we have shown a straightforward way in which the partial equilibrium model of Antràs and Chor (2013) can be embedded into a general equilibrium model with heterogeneity of final-good producers à la Melitz (2003). The resulting framework features supply chains that of endogenous length, with more productive final-good producers being served by longer supply chains. The firm-level mechanisms from the original Antràs-Chor model are completely unchanged in the general equilibrium setting developed in this paper, but they are complemented by a new set of mechanisms resulting from the effect of international trade in final goods on economy-wide factor allocation: Differences in the lengths of supply chains are magnified in the open economy, and this reallocation implies gains from trade that are strictly larger than in the original Melitz model.

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5 Appendix

5.1 Computing Average Profits $\overline{\pi}$

Average profits in the open economy can be written as

$$\overline{\pi} = \overline{\pi}_d + \chi\overline{\pi}_x$$

where $\overline{\pi}_d$ are average profits made by all firms in their home market, and $\overline{\pi}_x$ are average profits made by exporting firms in their export market. Using Eq. (26), $\overline{\pi}_x$ can be calculated as

$$\begin{aligned}\overline{\pi}_x &= \int_{\varphi_x^*}^{\infty} \pi_x(\varphi) \frac{g(\varphi)}{1 - G(\varphi_x^*)} d\varphi \\ &= \Theta\theta\beta(1-\beta)^{\frac{\rho}{1-\rho}} \Omega A \left[M_x(\varphi_x^*)\right]^{\frac{s\rho}{1-\rho}} (\varphi_x^*)^{\frac{\rho}{1-\rho}} - f_x\end{aligned}$$

Using Eqs. (26), (27), and the exporter indifference condition

$$\beta(1-\beta)^{\frac{\rho}{1-\rho}} \Omega A (\varphi_x^*)^{\frac{\rho}{1-\rho}} \left[M_d(\varphi_x^*)\right]^{\frac{s\rho}{1-\rho}} \left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right] = f_x,$$

we can rewrite this expression as

$$\overline{\pi}_x = \left[\frac{\Theta\theta(1+\theta)^{s\varepsilon}}{(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1} - 1 \right] f_x$$
$$= \Theta\theta(1+\theta)^{s\varepsilon}\xi^{\varepsilon} f - f_x$$

The average domestic profits $\overline{\pi}_d$ can be calculated as,

$$\begin{aligned}
\overline{\pi}_d &= \int_{\varphi_d^*}^{\infty} \pi(\varphi) \frac{g(\varphi)}{1-G(\varphi_d^*)} d\varphi \\
&= (\varphi_d^*)^k \beta (1-\beta)^{\frac{\rho}{1-\rho}} \Omega A \left\{ \int_{\varphi_d^*}^{\varphi_x^*} [M_d(\varphi)]^{\frac{s\rho}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}} g(\varphi) d\varphi + \int_{\varphi_x^*}^{\infty} [M_x(\varphi)]^{\frac{s\rho}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}} g(\varphi) d\varphi \right\} - f \\
&= (\varphi_d^*)^k \beta (1-\beta)^{\frac{\rho}{1-\rho}} \Omega A \left\{ \begin{array}{c} \int_{\varphi_d^*}^{\infty} [M_d(\varphi)]^{\frac{s\rho}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}} g(\varphi) d\varphi \\ + [(1+\theta)^{s\varepsilon} - 1] \int_{\varphi_x^*}^{\infty} [M_d(\varphi)]^{\frac{s\rho}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}} g(\varphi) d\varphi \end{array} \right\} - f \\
&= (\varphi_d^*)^k \beta (1-\beta)^{\frac{\rho}{1-\rho}} \Omega A \int_{\varphi_d^*}^{\infty} [M_d(\varphi)]^{\frac{s\rho}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}} g(\varphi) d\varphi \\
&\quad + [(1+\theta)^{s\varepsilon} - 1] \xi^{-k} (\varphi_x^*)^k \left\{ \beta (1-\beta)^{\frac{\rho}{1-\rho}} \Omega A \int_{\varphi_x^*}^{\infty} [M_d(\varphi)]^{\frac{s\rho}{1-\rho}} \varphi^{\frac{\rho}{1-\rho}} g(\varphi) d\varphi \right\} - f \\
&= \Theta f + [(1+\theta)^{s\varepsilon} - 1] \xi^{-k} \Theta \frac{f_x}{\left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1 \right]} - f \\
&= \Theta f + [(1+\theta)^{s\varepsilon} - 1] \xi^{-k} \Theta \xi^{\varepsilon} f - f
\end{aligned}$$

Substituting for $\overline{\pi}_d$, $\overline{\pi}_x$ and χ, we can derive

$$
\begin{aligned}
\overline{\pi} &= \overline{\pi}_d + \chi\overline{\pi}_x \\
&= \Theta f + \left[(1+\theta)^{s\varepsilon} - 1\right]\Theta\xi^{\varepsilon-k} f - f + \xi^{-k}\left[\Theta\theta(1+\theta)^{s\varepsilon}\xi^{\varepsilon} f - f_x\right] \\
&= (\Theta - 1)\, f + \left[(1+\theta)^{s\varepsilon} - 1 + \theta(1+\theta)^{s\varepsilon}\right]\Theta\xi^{\varepsilon-k} f - \xi^{-k} f_x \\
&= (\Theta - 1)\, f + \left[(1+\theta)^{s\varepsilon+1} - 1\right]\Theta\xi^{\varepsilon-k} f - \xi^{-k} f_x \\
&= (\Theta - 1)\, f + \left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right] f\Theta\xi^{\varepsilon-k} - \xi^{-k} f_x \\
&= (\Theta - 1)\, f + f_x\xi^{-\varepsilon}\Theta\xi^{\varepsilon-k} - \xi^{-k} f_x \\
&= (\Theta - 1)\, f + \Theta\xi^{-k} f_x - \xi^{-k} f_x \\
&= (\Theta - 1)\,(\chi f_x + f) \\
&= \overline{\pi}_a\left(1 + \chi\frac{f_x}{f}\right)
\end{aligned}
$$

5.2 Proof of Proposition 1

Combining Eqs. (29) and (31) gives

$$
\frac{M_x(\varphi)}{M_a(\varphi)} = (1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}}\left(1 + \chi\frac{f_x}{f}\right)^{-\frac{\varepsilon}{k}}
$$

Rewriting this expression gives

$$
\begin{aligned}
&\left[(1+\theta)^{-\frac{(1-\rho)k}{\rho}} + (1+\theta)^{-\frac{(1-\rho)k}{\rho}}\chi\frac{f_x}{f}\right]^{-\frac{\varepsilon}{k}} \\
&\qquad = \left[(1+\theta)^{-\frac{(1-\rho)k}{\rho}} + (1+\theta)^{-\frac{(1-\rho)k}{\rho}}\left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right]^{\frac{k}{\varepsilon}}\left(\frac{f_x}{f}\right)^{1-\frac{k}{\varepsilon}}\right]^{-\frac{\varepsilon}{k}}
\end{aligned}
$$

Notice that $\xi > 1$, which implies that

$$
\left(\frac{f_x}{f}\right) > \left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right]
$$

Denote $B(\theta, f_x, f) = (1+\theta)^{-\frac{(1-\rho)k}{\rho}} + (1+\theta)^{-\frac{(1-\rho)k}{\rho}} \left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right]^{\frac{k}{\varepsilon}} \left(\frac{f_x}{f}\right)^{1-\frac{k}{\varepsilon}}$, therefore

$$
\begin{aligned}
B(\theta, f_x, f) &< (1+\theta)^{-\frac{(1-\rho)k}{\rho}} + (1+\theta)^{-\frac{(1-\rho)k}{\rho}} \left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right]^{\frac{k}{\varepsilon}} \left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right]^{1-\frac{k}{\varepsilon}} \\
&= (1+\theta)^{-\frac{(1-\rho)k}{\rho}} + (1+\theta)^{-\frac{(1-\rho)k}{\rho}} \left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right] \\
&= (1+\theta)^{\frac{(1-\rho)}{\rho}(\varepsilon-k)} < 1
\end{aligned}
$$

This implies $M_x(\varphi)/M_a(\varphi) > 1$, which proves Proposition 1.

5.3 The Share of Expenditure on Domestically Produced Goods

Use λ_x to denote the share of expenditure going to imported products. Using our result from above that $\overline{\pi}_x = \Theta\theta(1+\theta)^{s\varepsilon}\xi^{\varepsilon} f - f_x$, since the two countries are symmetric, the average revenue of imported products is $\overline{r}_x = \frac{1}{\beta}\Theta\theta(1+\theta)^{s\varepsilon}\xi^{\varepsilon} f$. Then we have

$$
\lambda_x = \frac{\overline{r}_x \chi N}{L} = \frac{\frac{1}{\beta}\Theta\theta(1+\theta)^{s\varepsilon}\xi^{\varepsilon} f \chi \frac{\frac{\beta L}{\Theta f}}{1+\chi\frac{f_x}{f}}}{L} = \frac{\theta(1+\theta)^{s\varepsilon}\xi^{\varepsilon}\chi}{1+\chi\frac{f_x}{f}}
$$

The share of domestic expenditure follows as

$$
\lambda_d = 1 - \lambda_x = \frac{1 + \chi\left[\frac{f_x}{f} - \theta(1+\theta)^{s\varepsilon}\xi^{\varepsilon}\right]}{1+\chi\frac{f_x}{f}},
$$

and we can show that if $s > 0$,

$$
\frac{f_x}{f} - \theta(1+\theta)^{s\varepsilon}\xi^{\varepsilon} = \frac{f_x}{f} - \frac{f_x\theta(1+\theta)^{s\varepsilon}}{f\left[(1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}} - 1\right]} = \frac{f_x}{f}\left[\frac{(1+\theta)^{s\varepsilon} - 1}{(1+\theta)^{s\varepsilon+1} - 1}\right] > 0
$$

Therefore

$$\lambda_d > \frac{1}{1+\chi\frac{f_x}{f}}$$

The gains from the trade would be underestimated if we use the share of domestic expenditure as a sufficient statistic, conditional on observed trade data.

5.4 Additional Calculations from Section 3

Solving program (33), we obtain the optimal choice of labour employment:

$$l\left[m,\beta\left(m\right)\right] = \left[\left(1-\beta\left(m\right)\right)\rho\left(A^{1-\rho}\varphi^{\rho}M^{(s+1-1/\alpha)\rho}\right)^{\frac{\alpha}{\rho}}r\left(M,m,\varphi\right)^{\frac{\rho-\alpha}{\rho}}\right]^{\frac{1}{1-\alpha}}$$

Plugging this expression into $r'\left(M,m,\varphi\right)$ delivers the following differential equation:

$$r'\left(M,m,\varphi\right) = \frac{\rho}{\alpha}\left(A^{1-\rho}\varphi^{\rho}M^{(s+1-1/\alpha)\rho}\right)^{\frac{\alpha}{\rho(1-\alpha)}}r\left(M,m,\varphi\right)^{\frac{\rho-\alpha}{\rho(1-\alpha)}}\left[\rho\left(1-\beta\left(m\right)\right)\right]^{\frac{\alpha}{1-\alpha}}$$

We can verify that the solution to this differential equation is given by:

$$\begin{aligned} r\left(M,m,\varphi\right) =& A\left(\varphi M^{s+1-1/\alpha}\right)^{\frac{\rho}{1-\rho}}\left(\frac{1-\rho}{1-\alpha}\right)^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}}\rho^{\frac{\rho}{1-\rho}}\left[\int_0^m\left(1-\beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}}di\right]^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} \\ =& \Omega A\left(\varphi M^{s+1-1/\alpha}\right)^{\frac{\rho}{1-\rho}}\left[\int_0^m\left(1-\beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}}di\right]^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} \end{aligned}$$

Differentiating $r\left(M,m,\varphi\right)$ and substituting into $\int_0^M\beta\left(i\right)r'\left(M,i,\varphi\right)di$, we can compute operating profits as:

$$\pi_{op} = \frac{\rho\left(1-\alpha\right)}{\alpha\left(1-\rho\right)}\Omega A\left(\varphi M^{s+1-1/\alpha}\right)^{\frac{\rho}{1-\rho}}\int_0^M\beta\left(i\right)\left(1-\beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}}\left[\int_0^i\left(1-\beta\left(u\right)\right)^{\frac{\alpha}{1-\alpha}}du\right]^{\frac{\rho-\alpha}{\alpha(1-\rho)}}di$$

Following the analysis in Antràs and Chor (2013) and Alfaro et al. (2016), we know that for a final-good producer φ served by a supply chain of length M there exist thresholds $m_c^* \in (0,M]$ and $m_s^* \in (0,M]$ separating the stages that are outsourced from those that are integrated in the complements case and the substitutes case, respectively. With sequential complements, all production stages before m_c^* are outsourced, with the remainder being integrated, while in the case of sequential substitutes all stages before m_s^* are integrated, with the remainder being outsourced.

Consider first the complements case, we can rewrite the operating profit π_{op} as

$$\pi_{op} = \Omega A \left(\varphi M^{s+1-1/\alpha}\right)^{\frac{\rho}{1-\rho}} \int_0^M \beta(i) \frac{\partial \left(\left[\int_0^i (1-\beta(u))^{\frac{\alpha}{1-\alpha}} du \right]^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} \right)}{\partial i} di,$$

in which

$$\begin{aligned}
&\int_0^M \beta(i) \frac{\partial \left(\left[\int_0^i (1-\beta(u))^{\frac{\alpha}{1-\alpha}} du \right]^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} \right)}{\partial i} di \\
&= \int_0^{m_c} \beta(i) \frac{\partial \left(\left[\int_0^i (1-\beta(u))^{\frac{\alpha}{1-\alpha}} du \right]^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} \right)}{\partial i} di + \int_{m_c}^M \beta(i) \frac{\partial \left(\left[\int_0^i (1-\beta(u))^{\frac{\alpha}{1-\alpha}} du \right]^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} \right)}{\partial i} di \\
&= \beta_o (1-\beta_o)^{\frac{\rho}{1-\rho}} m_c^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} + \beta_v \left(\begin{array}{c} \left[m_c (1-\beta_o)^{\frac{\alpha}{1-\alpha}} + (M-m_c)(1-\beta_v)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} \\ - \left[m_c (1-\beta_o)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{\rho(1-\alpha)}{\alpha(1-\rho)}} \end{array} \right)
\end{aligned}$$

Taking the first-order condition with respect to m_c, we find:

$$(\beta_v - \beta_o)(1-\beta_o)^{\frac{\rho}{1-\rho}} = \beta_v \left[(1-\beta_o)^{\frac{\alpha}{1-\alpha}} - (1-\beta_v)^{\frac{\alpha}{1-\alpha}} \right] \left((1-\beta_o)^{\frac{\alpha}{1-\alpha}} + \frac{M-m_c^*}{m_c^*} (1-\beta_v)^{\frac{\alpha}{1-\alpha}} \right)^{\frac{\rho-\alpha}{\alpha(1-\rho)}}$$

From this, we obtain

$$\frac{m_c^*}{M} = h_c^* = \left\{ 1 + \left(\frac{1-\beta_o}{1-\beta_v} \right)^{\frac{\alpha}{1-\alpha}} \left[\left(\frac{1-\frac{\beta_o}{\beta_v}}{1-\left(\frac{1-\beta_o}{1-\beta_v}\right)^{-\frac{\alpha}{1-\alpha}}} \right)^{\frac{\alpha(1-\rho)}{\rho-\alpha}} - 1 \right] \right\}^{-1} \doteq \Psi_c(\beta_v, \beta_o, \alpha, \rho)$$

The threshold in the substitutes case can be derived in an analogous way. As a result, m_s^* is given by

$$\frac{m_s^*}{M} = h_s^* = \left\{ 1 + \left(\frac{1-\beta_v}{1-\beta o} \right)^{\frac{\alpha}{1-\alpha}} \left[\left(\frac{\frac{\beta_v}{\beta o} - 1}{\left(\frac{1-\beta_v}{1-\beta o} \right)^{-\frac{\alpha}{1-\alpha}} - 1} \right)^{\frac{\alpha(1-\rho)}{\rho-\alpha}} - 1 \right] \right\}^{-1} \doteq \Psi_s \left(\beta_v, \beta_o, \alpha, \rho \right)$$

Plugging the expression of m_k^* with $k = c, s$ into Eq. (34), we can rewrite the total revenue along a supply chain:

$$\begin{aligned} r\left(M, \varphi\right) &= \Omega A \left(M^{(s+1-1/\alpha)} \varphi \right)^{\frac{\rho}{1-\rho}} \left[\int_0^{h_k^* M} \left(1 - \beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}} di + \int_{h_k^* M}^{M} \left(1 - \beta\left(j\right)\right)^{\frac{\alpha}{1-\alpha}} dj \right]^{\frac{\rho(1-\alpha)}{(1-\rho)\alpha}} \\ &= \Omega A \left(M^{(s+1-1/\alpha)} \varphi \right)^{\frac{\rho}{1-\rho}} \left[h_k^* M \left(1 - \beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}} + \left(M - h_k^* M\right) \left(1 - \beta\left(j\right)\right)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{\rho(1-\alpha)}{(1-\rho)\alpha}} \\ &= \Omega A \left(M^s \varphi \right)^{\frac{\rho}{1-\rho}} \left[h_k^* \left(1 - \beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}} + \left(1 - h_k^*\right) \left(1 - \beta\left(j\right)\right)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{\rho(1-\alpha)}{(1-\rho)\alpha}} \\ &= \Omega A \left(M^s \varphi \right)^{\frac{\rho}{1-\rho}} \Lambda_k \left(\beta_v, \beta_o, \alpha, \rho \right) \end{aligned}$$

with $\beta(i) = \beta_o$ and $\beta(j) = \beta_v$ in the complements case, and with $\beta(i) = \beta_v$ and $\beta(j) = \beta_o$ in the substitutes case. Λ_k is another parameter collecting terms involving β_v and β_o.

From the expression of $r(M, m, \varphi)$, we have

$$r'\left(M, m, \varphi\right) = \frac{\rho\left(1-\alpha\right)}{\alpha\left(1-\rho\right)} \Omega A \left(\varphi M^{s+1-1/\alpha} \right)^{\frac{\rho}{1-\rho}} \left[\int_0^m \left(1 - \beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}} di \right]^{\frac{\rho-\alpha}{\alpha(1-\rho)}} \left(1 - \beta\left(m\right)\right)^{\frac{\alpha}{1-\alpha}}$$

therefore we can rewrite $r'(M, m, \varphi)$ in terms of h as the following: if $\frac{m}{M} = h > h_k^*$,

$$r'\left(h, M, \varphi\right) = \frac{\rho\left(1-\alpha\right)}{\alpha\left(1-\rho\right)} \Omega A \varphi^{\frac{\rho}{1-\rho}} M^{\frac{1-(s+1)\rho}{\rho-1}} \left(1 - \beta\left(j\right)\right)^{\frac{\alpha}{1-\alpha}} \left[h_k^* \left(1 - \beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}} + \left(h - h_k^*\right) \left(1 - \beta\left(j\right)\right)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{\rho-\alpha}{\alpha(1-\rho)}}$$

and if $\frac{m}{M} = h < h_k^*$,

$$r'\left(h, M, \varphi\right) = \frac{\rho\left(1-\alpha\right)}{\alpha\left(1-\rho\right)} \Omega \varphi^{\frac{\rho}{1-\rho}} M^{\frac{1-(s+1)\rho}{\rho-1}} \left(1 - \beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}} \left[h_k^* \left(1 - \beta\left(i\right)\right)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{\rho-\alpha}{\alpha(1-\rho)}}$$

Since both suppliers' profits and wage costs are proportional to $r'(h, M, \varphi)$, we have

$$\frac{M(\varphi_1)}{M(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\varepsilon},$$

and therefore the length of supply chains is increasing in φ with the same constant elasticity as in the benchmark model. Plugging the expression of m_k^* with $k = c, s$ into the final-good producer's operating profits π_{op}, we can show that $\pi_{op} = \Gamma_k(\beta_v, \beta_o, \alpha, \rho)\,\Omega A\,(M^s\varphi)^{\frac{\rho}{1-\rho}}$ with $k = c, s$, in which $\Gamma_k(\beta_v, \beta_o, \alpha, \rho)$ collects β_v and β_o. The profit of a final-good producer with productivity φ is given by

$$\pi(M, \varphi) = \Gamma_k(\beta_v, \beta_o, \alpha, \rho)\,\Omega A\,(M^s\varphi)^{\frac{\rho}{1-\rho}} - f$$

In analogy to the benchmark model, suppliers' ex-ante average profits along a supply chain are given by $(1-\alpha)\left[r(M, \varphi) - \pi_{op}\right]/M$, which are equalised across supply chains in equilibrium:

$$\frac{(1-\alpha)(\Lambda_k - \Gamma_k)\,\Omega A\left[M(\varphi_1)^s\varphi_1\right]^{\frac{\rho}{1-\rho}}}{M(\varphi_1)} = \frac{(1-\alpha)(\Lambda_k - \Gamma_k)\,\Omega A\left[M(\varphi_2)^s\varphi_2\right]^{\frac{\rho}{1-\rho}}}{M(\varphi_2)} = f_m$$

The zero cutoff profit condition $\pi_{op} = f$ follows as

$$\Omega A\left[M(\varphi^*)^s\varphi^*\right]^{\frac{\rho}{1-\rho}} = \frac{f}{\Gamma_k}$$

Immediately,

$$M(\varphi^*) = \frac{f(1-\alpha)(\Lambda_k - \Gamma_k)}{f_m\Gamma_k}$$

and the average length of supply chains follows as $\overline{M} = \Theta M(\varphi^*)$. As in the benchmark model, the cutoff productivity φ^* is given by

$$\varphi^* = \left[\frac{(\Theta-1)f}{f_e}\right]^{\frac{1}{k}}$$

The average revenue of final-good producers equals $\Theta f \Lambda_k / \Gamma_k$, the number of final-good producers is given by

$$N = \frac{\Gamma_k L}{\Lambda_k \Theta f},$$

and the total number of suppliers follows as

$$\overline{M} N = \frac{(1-\alpha)(\Lambda_k - \Gamma_k)}{\Lambda_k f_m} L.$$

Using the zero profit condition for final-good producers, welfare per worker is given by

$$W = P^{-1} = \left(\frac{\Gamma_k \Omega L}{f} \right)^{\frac{1-\rho}{\rho}} \left[M(\varphi^*) \right]^s \varphi^*$$

In the open economy, two additional costs are involved. An exporting firm has the total revenue that is a multiple $1+\theta$ of its domestic revenue. International trade has no effect on the threshold relative position h_k^*. In analogy to Eq. (25), we rewrite the indifference condition for the marginal exporting firm as:

$$(1+\theta)\Gamma_k(\beta_v, \beta_o, \alpha, \rho)\Omega A \left[M_x(\varphi_x^*)^s \varphi_x^* \right]^{\frac{\rho}{1-\rho}} - \Gamma_k(\beta_v, \beta_o, \alpha, \rho)\Omega A \left[M_d(\varphi_x^*)^s \varphi_x^* \right]^{\frac{\rho}{1-\rho}} = f_x$$

Similarly, we can also show that the cross-regime SIC implies

$$\frac{M_x(\varphi)}{M_d(\varphi)} = (1+\theta)^{\frac{\varepsilon(1-\rho)}{\rho}},$$

exactly as in the benchmark model. As a consequence, we find that many results from this model are still valid in the extension discussed here. In particular, the share of exporting firms χ is the same as before, and hence the gains from trade W/W_a in this extension are the same as in our benchmark model.